THE KALAMADIS EXPERIMENT
Nick Herbert (quanta@cruzio.com)

ABSTRACT: I examine an original proposal by Demetrios Kalamidas for using quantum entanglement to send superluminal signals from Bob to Alice. I show, using my own version of the Kalamidas experiment, that no FTL signaling actually exists by explicit calculation of the photon correlations at Alice's detectors with the readings of Bob's detectors in all possible Kalamidas configurations.

"We have to remember that what we observe is not nature herself, but nature exposed to our method of questioning."  
—Werner Heisenberg

"If questions of a different kind can be asked, then Nature will respond in a new language."  
—Beverly Rubik

Quantum thinking is more than a century old and while we now possess a theory which (so far) describes exactly all experiments we can imagine, we still have not been able to visualize what sort of quantum reality underlies the quantum experiments, the results of which quantum theory predicts so well.

Quantum reality responds differently depending on what sorts of questions we ask of it but, for a single state of affairs, the sum of all the answers we get do not resolve into a agreed-upon conceptual picture. The situation resembles the story of the blind men and the elephant — each asking a different question of the animal. Quantum mechanics resembles that story except for the fact that we can't picture the elephant. On the other hand we possess
a wonderful theory that will predict the correct answer to any question you are able to ask about this invisible beast.

Recently American physicist Demetrios Kalamidas has come up with a new question to ask a particularly simple quantum system — a pair of path-entangled optical photons. All of the optical components that are needed to ask the "Kalamidas question" are available in modern optics labs but have never been assembled together in this particular way. The purpose of this paper is to calculate the result that nature will give to this new question and to verify or refute the surmise that the Kalamidas experiment might be used as a faster-than-light signaling device.

![FIG 1](image)

FIG 1 represents the path-entangled source S. Two photons A and B are emitted from the source in two possible back-to-back directions. Photon path A1 (going to the left) is correlated with photon path B1 (going to the right). A similar correlation exists twixt paths A2 and B2.
Anticipating using this system to signal between two parties, we refer to the A photon as "Alice's photon" and the B photon as "Bob's photon." Although there are two possible paths in each direction, there is only one photon going to each of the participants. Also since the strength of the quantum correlation does not fall off with distance, Alice and Bob could be light-years apart and the calculated results would not be different.

We write the product state $|A1\rangle|A2\rangle|B1\rangle|B2\rangle$ and symbolize "full path" (one photon) by $|1\rangle$ and "empty path (no photon) as $|0\rangle$. Using this convention the entangled state $|AB\rangle$ is written:

$$|AB\rangle = s^{-1}\{|1\rangle|0\rangle|1\rangle|0\rangle \exp(iQ) + |0\rangle|1\rangle|0\rangle|1\rangle\} \quad (1)$$

where $s = \sqrt{2}$ and $\exp(iQ)$ is a phase shift $Q$.

Here I make use of the marvelously efficient Dirac bra/ket notation. In Dirac's original book, the "ket" $|A\rangle$ represents a vector in Hilbert space; the "bra" $<A|$ represents that vector's partner in the dual Hilbert space and $<n|A>$ represents a wavefunction (in the number representation).

Most modern textbooks use a "shorthand Dirac notation", in which the ket $|A\rangle$ represents the wavefunction (or probability amplitude). And the "bra" $<A|$ represents that wavefunction's complex conjugate. In this new notation, the bra-ket product $<A|A>$ represents the probability that event A will actually happen.
On Alice's side of the source we place a simple interferometer in which Alice's two paths A1 and A2 are brought together in a 50/50 beam splitter. Alice's original photon paths are mixed and emerge as paths A3 and A4. Photon detectors are placed in each of these paths.

Loosely speaking, Alice's interferometer will display interference (a sinusoidal variation of photon counting rate at Detectors A3 and A4 as Alice varies phase Q) only if Alice's (A) photon "takes both paths at once".

If the experimental arrangement is such that it reveals which path Alice's photon took, then no interference will occur—Alice's Detector count rates will be independent of the phase Q.

A entanglement-powered superluminal signaling scheme would work by Bob asking two kinds of questions of his photon B. If that question preserves "which-path info" at A, then Alice will observe interference. If Bob's question "erases which-path info" at B, then interference will vanish. If we call Alice's interference a "1" and her non-interference a "0", then Bob by his two actions on his B photon can instantly send a message in binary code to Alice no matter how far apart he and she are separated.

I examine three kinds of questions that Bob might ask—three questions which either preserve or erase Alice's which-path information. They are the "Fock question", the "Frost question" and the "Kalamidas question". The first two questions (except for the names I have given these
questions) are familiar to most physicists. But the Kalamidas question is entirely new.

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MAKING THE FOCK CHOICE

FIG 2 illustrates Bob's "asking the Fock question" or "making the Fock choice". In this case Bob merely puts a photon detector in each of his two possible paths B1 and B2. If his B photon is detected in Detector B1 then he knows with certainty that Alice's photon took path A1. Likewise if Bob detects his photon in Detector B2.

I call this choice the "Fock choice" after Russian physicist
Vladimir Fock who gave his name to light that possesses a definite number of photons. "Fock states" are symbolized by |0>, |1>, |2>, etc, each ket labeled by how many photons are present in that quantum state.

Light commonly consists of an uncertain number of photons. Even laser light has an uncertain photon number. Fock light is therefore a very unnatural form of light. But Fock light can be produced in the lab. For instance by using a two-photon entangled state such as described by EQ (1).

When Bob makes the Fock choice on Source S—and measures a B photon in Detector B1, he knows that Fock light of form |1> (exactly one photon) is present in Alice's A1 path. And he also knows that Fock light of form |0> (exactly zero photons) is present in Alice's A2 path.

I now calculate the behavior of Fock light in Alice's interferometer. Since the other two questions involve beam splitters that are not 50/50, I will calculate here the behavior of photons at a generalized lossless beam splitter with transmission amplitude "t" and reflection amplitude "r". The probability for a photon to be either reflected or transmitted must be 1, requiring $t^2 + r^2 = 1$.

I will symbolize "$a^\dagger$" as the photon creation operator such that $|1> = a^\dagger |0>$, $|2> = a^\dagger a^\dagger |0>$. And, in general, $a^\dagger |n> = \sqrt{n+1} |n+1>$.

For the operation of the beam splitters on the photon creation operators $a^\dagger$ I choose the convention:
\[ a_3^\dagger = t a_1^\dagger - r a_2^\dagger \quad a_1^\dagger = t a_3^\dagger + r a_4^\dagger \quad (2) \]
\[ a_4^\dagger = r a_1^\dagger + t a_2^\dagger \quad a_2^\dagger = -r a_3^\dagger + t a_4^\dagger \]

Using these relations we can calculate the following useful relationships between beam-splitter input state \(|A_1>|A_2>|\) and beam-splitter output state \(|A_3>|A_4>|\):

<table>
<thead>
<tr>
<th>IN:</th>
<th>OUT:</th>
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<tbody>
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We see from EQ (3) that "nothing in" to both input ports of a beam splitter leads to "nothing out". EQ (3) expresses the result of zero photons on a beam splitter.

From EQ (4) we see that a single photon \(|1>|\) into one input port and "nothing" into the other input port results in a single photon \(|0>|\) entangled with the vacuum \(|0>|\) with various amplitudes depending on the beam splitter parameters. EQ (4) expresses the results of one photon on a beam splitter.

From EQ (5) we see that one photon \(|1>|\) inserted into each input port results in three possible output states
|2> |0>, |1> |1> and |0> |2> with amplitudes dependent on the beam splitter parameters. In the special case of a 50/50 beam splitter where \( t = r = s^{-1} \), the |1> |1> amplitude vanishes leaving only two terms. In this situation both photons end up in the same detector—there is no cross term. This situation is an example of "two-photon interference"—also called the Hong-Ou-Mandel effect. EQ (5) expresses the results of two photons on a beam splitter.

Each of these equations will be useful for calculating the results of three questions asked by Bob concerning the nature of two-photon entangled state described by EQ (1).

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CALCULATING THE FOCK CHOICE

Referring to FIG 2 we begin with the entangled state of EQ (1) and use a 50/50 beam splitter to transform the state |A1> |A2> into the state |A3> |A4>. For the 50/50 case the one photon beam splitter relations simplify to:

\[
|1> |0> \rightarrow s^{-1}(|1> |0> - |0> |1>) \tag{6}
\]

\[
|0> |1> \rightarrow s^{-1}(|1> |0> + |0> |1>) \tag{7}
\]

|A1> \rightarrow s^{-1}(|A3> - |A4>)

|A2> \rightarrow s^{-1}(|A3> + |A4>)

Bob's photon paths B1 and B2 are still entangled with Alice's photon paths. But instead of being entangled with paths A1 and A2, they are now entangled with Alice's photon paths A3 and A4.
\[ |AB>_{\text{FO}} = \frac{1}{2} \{ (\exp iQ|A3> - \exp iQ|A4>)|B1> \\
+ (|A3> + |A4>)|B2> \} \]  

(8)

\[ = \frac{1}{2} \{ (\exp iQ|B1> + |B2>)|A3> - (\exp iQ|B1> - |B2>)|A4> \} \]

This equation (EQ (8)) for the action of Alice's interferometer on Alice's original photon paths A1, A2 will be used for all three choices—Fock, Frost and Kalamidas. In all three cases Alice asks the same question of Source S. But Bob will ask three different questions of his photon B in an attempt to signal to Alice by inducing interference in her detectors or to destroy it.

When Bob makes the Fock choice, he asks the question: did my B photon go along path B1, leaving path B2 empty (symbolized by \([1><0]_B\))? Or did my B photon end up in the state symbolized by \([0><1]_B\)?

Expanding EQ (8) to include Bob's two possible outcomes, we obtain:

\[ |AB>_{\text{FO}} = \frac{1}{2} \{ (\exp iQ[10]_B + [01]_B)|A3> \\
- (\exp iQ[10]_B - [01]_B)|A4> \} \]  

(9)

For this Fock choice there are four possible outcomes (each mutually distinguishable):

\[ [10]_A[10]_B \quad [10]_A[01]_B \quad [01]_A[10]_B \quad [01]_A[01]_B \]

with probability amplitudes:
To evaluate the probability of these 4 outcomes we invoke the Feynman rule: if the results are distinguishable, sum amplitudes before squaring; if the results are not distinguishable, square before summing.

Formally we evaluate probabilities by multiplying the bra \( \langle AB| \) into the ket \( |AB> \).

\[
\langle AB|AB> = \frac{1}{4} \left\{ \langle A3|A3\rangle_B[10] + \langle A3|A3\rangle_B[01] + \langle A4|A4\rangle_B[10] + \langle A4|A4\rangle_B[01]\right\} \tag{9}
\]

Separating out these four distinguishable processes, we see from EQ (9) that these 4 actualities have the same probability. And that there is no interference anywhere.

\[
\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad (10)
\]

The total probability adds up to 1. The phase factor \( Q \) vanishes because \( | \exp iQ |^2 = 1 \). Therefore Alice sees no interference. The probability for Alice to detect a photon at Detector A3 is 1/2, which is the same as her probability to detect a photon at Detector A4. What Alice observes, when Bob makes the Fock choice, is a sequence of counts, randomly distributed between her two detectors, similar to what she might observe when flipping a fair coin—an utterly random distribution of Heads and Tails.
MAKING THE FROST CHOICE

Bob (B) asks the “Frost Question”:
If photon takes both paths,
which path is longer (phase difference)?
Since “which-path info” is erased,
Alice (A) measures (B - conditional) interference.

FIG 3

"Two roads diverged in a yellow wood," mused poet
Robert Frost, "And sorry I could not travel both/ And be
one traveler." Robert Frost could not take both paths at the
same time—and still be one traveler—but a single photon
can. A single photon can take both paths—and still be one
traveler. And when that photon rejoins itself, it can
produce interference fringes (but only if Alice looks for
them) whose pattern depends on the relative lengths of
each path. If Bob erases which-path information on his
side of the experiment and allows his B photon to take
both paths, will his act (making the Frost choice) cause
Alice’s A entangled photon to take both paths and produce
visible fringes in her interferometer?
The answer is yes.

Bob makes the Frost choice by using beam splitter \( BS_B \) to combine both paths of his B photon to ambiguates B's which-path information.

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CALCULATING THE FROST CHOICE

From Eq (4) we have for the generalized beam splitter:

\[
\text{IN: } |1\rangle|0\rangle \rightarrow \text{OUT: } t|1\rangle|0\rangle - r|0\rangle|1\rangle \quad (4)
\]

\[
\text{IN: } |0\rangle|1\rangle \rightarrow \text{OUT: } r|1\rangle|0\rangle + t|0\rangle|1\rangle
\]

\[
|B_1\rangle \rightarrow t|B_3\rangle - r|B_4\rangle \quad (11)
\]

\[
|B_2\rangle \rightarrow r|B_3\rangle + t|B_4\rangle
\]

We use Eq (8) for the action of Alice's interferometer:

\[
|AB\rangle_{FR} = \quad (8)
\]

\[
= \frac{1}{2} \{(\exp \text{i}Q|B_1\rangle + |B_2\rangle)|A_3\rangle - (\exp \text{i}Q|B_1\rangle - |B_2\rangle)|A_4\rangle\}
\]

Next we insert Eq (10) onto Eq (8), adding a phase shift \( \exp \text{i}P \) in the B1 path for generality.

\[
|AB\rangle_{FR} = \frac{1}{2}\{(\exp \text{i}(Q+P)t + r) |B_3\rangle|A_3\rangle \\
+(- \exp \text{i}(Q+P)t + r) |B_3\rangle|A_4\rangle \\
+(- \exp \text{i}(Q+P)r + t) |B_4\rangle|A_3\rangle \\
(\exp \text{i}(Q+P)r + t) |B_4\rangle|A_4\rangle\} \quad (12)
\]
EQ (12) is all we need to calculate anything we want to know about the outcomes of the Frost choice. Alice's interferometer uses 50/50 beam splitters but Bob's interferometer can have any value for transmission "t". EQ (12) expresses the amplitudes for four different outcomes—for instance the amplitude corresponding to outcome \(|B3\rangle||A3\rangle\) in which Bob's photon is detected in Bob's Detector B3 in coincidence with Alice detecting a photon in Alice's Detector A3.

We now calculate the probabilities for these four outcomes by multiplying bra \( _{AB} \langle AB | \rangle_{FR} \) into ket \( |AB\rangle_{FR} \). For conciseness we define \( R = Q + P \).

\[
{<AB|AB>}_{FR} = \frac{1}{4} \left\{ (1 + 2rt \cos R) \langle B3|B3\rangle\langle A3|A3\rangle + (1 - 2rt \cos R) \langle B3|B3\rangle\langle A4|A4\rangle + (1 - 2rt \cos R) \langle B4|B4\rangle\langle A3|A3\rangle + (1 + 2rt \cos R) \langle B4|B4\rangle\langle A4|A4\rangle \right\}
\]

To check our calculations we specialize EQ (13) to the case where \( t = 1 \). In this case the beam splitter is transparent and no mixing of paths occur. Hence which-path information is fully present. Using EQ (13) it is easy to see that, for \( t = 0 \), the Frost choice becomes the Fock choice with outcomes corresponding to EQ (10)—no interference; all four outcomes equiprobable.

Now we specialize to the case in which Bob uses a 50/50 beam splitter to mix his B photons so as to maximally erase which-path information.

In this case \( t = r = s^{-1} \), and using some trig identities, the
probability for all 4 outcomes simplifies to:

\[
\langle ABIAB \rangle_{FR} = \frac{1}{2} \left\{ \cos^2 R/2 \langle B3|B3\rangle \langle A3|A3\rangle + \sin^2 R/2 \langle B3|B3\rangle \langle A4|A4\rangle + \sin^2 R/2 \langle B4|B4\rangle \langle A3|A3\rangle + \cos^2 R/2 \langle B4|B4\rangle \langle A4|A4\rangle \right\}
\]

(14)

It is easy to see from EQ (14) that the probabilities all sum to one. It is also obvious that adding the probabilities for Alice's \( \langle A3|A3\rangle \) outcome, we get no interference.

What happened? Supposedly Bob's erasing his which-path information should produce interference at Alice's detectors. And indeed it does. But it is a special kind of interference which depends on coincidence detection. Coincidence detection, for instance, between Bob's outcome \( \langle B3|B3\rangle \) and Alice's outcome \( \langle A3|A3\rangle \).

From EQ (14) we see that the probability of this joint counting event is \( \frac{1}{2} \cos^2 R/2 \).

Thus interference fringes at Alice's detectors are indeed produced by Bob's Frost choice. But Alice's fringes are conditional on a coincidence signal from one or more of Bob's detectors. Since these signals can at best only be sent from Bob to Alice at light speed or slower, the Frost choice does not seem to offer an opportunity for entanglement-based superluminal signaling.
If Alice looks at her Detector A3 only when she receives a coincidence trigger from Bob's Detector B3, she observes a \(1/2 \cos^2 R/2\) fringe signal. If Alice looks at Detector A3 only when she receives a coincidence trigger from Bob's Detector B4, she observes a \(1/2 \sin^2 R/2\) "anti-fringe" signal. Added together, (corresponding to looking at all photons without coincidence triggering) the antifringes exactly fill in the gaps of the fringes and Alice's interference signal vanishes.

This result was obtained for the special case of 50/50 beam splitter. To calculate the General Frost Effect we define \(F = 2rt\) where \(F\) can vary from 0 to 1, depending on the beam splitter transmission/reflection parameters.

With this substitution EQ (13) becomes:

\[
<\text{ABIAB}_{\text{FR}} = 1/4 \left\{ (1 + F \cos R) <B3|B3><A3|A3> \\
+ (1 - F \cos R) <B3|B3><A4|A4> \right. \\
+ \left. (1 - F \cos R) <B4|B4><A3|A3> \\
+ (1 + F \cos R) <B4|B4><A4|A4> \right\}
\]

which becomes:

\[
<\text{ABIAB}_{\text{FR}} = 1/4 \left\{ (1 - F + 2F(\cos^2 R/2)) <B3|B3><A3|A3> \\
(1 - F + 2F(\sin^2 R/2)) <B3|B3><A4|A4> \\
(1 - F + 2F(\sin^2 R/2)) <B4|B4><A3|A3> \\
(1 - F + 2F(\cos^2 R/2)) <B4|B4><A4|A4> \right\}
\]

which shows that, as the beam splitter parameter is varied from 0 to 1, there is a continuous transition between the Fock choice \((F = 0)\) and an "impure" Frost choice consisting in an incoherent component \(((1 - F)/2)\) and a
coherent component (F/2) (which represents the magnitude of Alice's "coincidence-triggered interference". When F = 1 (the case of a 50/50 beam splitter), the incoherent term becomes zero and the magnitude of Alice's (coincidence-triggered) interference is maximized.

The above conclusions for the Fock choice and for the Frost choice are well-known—except for the names, which I have invented. I present these results only as a warmup for calculating the probability of Alice's A3/A4 outcomes when Bob makes the Kalamidas choice. Unlike the Fock and Frost choices, the Kalamidas choice is new.

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MAKING THE KALAMIDAS CHOICE

Bob (B) asks the “Kalamidas Question”: If I erase “which-path info” in a new way what sort of interference will Alice (A) observe?

The Kalamidas choice erases which-path information about photon B using an ingenious scheme invented by
Demetrios Kalamidas, a physics graduate of CCNY in New York.

Kalamidas proposes to erase the which-path info about Bob's photons encoded in definite-number Fock states by a process I call "ambiguification". This ambiguification is carried out by mixing the number-certain Fock states with a number-uncertain "gray state".

Kalamidas's clever method of photon ambiguification is the unique and essential feature of his new superluminal signaling scheme.

The Gray Photon State $|U> \text{ is defined:}$

$$|U> = x|0> + y|1> \quad (15)$$

Unlike a Fock state which contains a definite number of photons, say zero, one, two, the U state contains an uncertain number -- between zero and 1. When you measure a "gray photon" sometimes you get one photon; sometimes you get none.

While a Fock state can only be Full ($|B> = |0>$) or Empty ($|B> = |0>$), the U state can be "half-full". The U state is the most general number-uncertain 1-photon state and its "grayness" varies from empty to full depending on the coefficient $x$. To normalize this state we require $x^2 + y^2 = 1$.

To quantum-erase B-path information, Kalamidas mixes each of his B-photon modes with a gray photon U in an asymmetric beam splitter characterized by transmission
amplitude t and reflection amplitude r where \( t^2 + r^2 = 1 \).

\[
\begin{align*}
|B1\rangle & \& |U1\rangle \implies |B3\rangle & \& |B4\rangle \\
|B2\rangle & \& |U2\rangle \implies |B5\rangle & \& |B6\rangle
\end{align*}
\]

action of beam splitters #1 and #2

In beam splitter #1 the \(|B1\rangle\) mode is mixed with the \(|U1\rangle\) mode to produce the output modes \(|B3\rangle\) and \(|B4\rangle\). Similarly with beam splitter #2. Detectors sensitive to photon number are placed at the outputs of these beam splitters. These detectors are labeled B3, B4, B5 and B6.

The outputs of these four detectors constitute the "signal" of the Kalamidas device and will be symbolized thus:

\[[B3,B4,B5,B6]\]

For the Kalamidas device there are 15 possible outcomes — the amplitudes of which I now proceed to calculate.

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CALCULATING THE KALAMIDAS CHOICE

Beam splitter #1 transforms \(|B1\rangle\) into \(|B3\rangle|B4\rangle\)
Beamsplitter #2 transforms \(|B2\rangle\) into \(|B5\rangle|B6\rangle\)

so that a single \(|B\rangle\) goes into a product of Bs: \(|B\rangle|B\rangle\)

Since the splitters and \(|U\rangle\) are identical so are the \(|B\rangle\) transformations.
There are two kinds of \( |B> \) transformations depending on whether the initial \( |B> \) was Full (\( |B> = |1> \)) or Empty (\( |B> = |0> \)).

For instance \( |B_1> \text{ Full} \implies [|B_3>|B_4>] \text{ Full} \) or \( [|B_3>|B_4>] \text{ F} \) and \( |B_1> \text{ Empty} \implies [|B_3>|B_4>] \text{ E} \)

Since all conditions are the same we only have to calculate two expressions:

\[
|B> \text{ F} \implies [|B>|B>] \text{ F} \quad \text{and} \quad |B> \text{ E} \implies [|B>|B>] \text{ E}
\]

The results are exactly the same for photons B1 and B2.

An Empty B_{\text{IN}} mode is transformed by the beamsplitter into three "Empty output modes":

\[
[|B_3>|B_4>] \text{ E} = [|B_5>|B_6>] \text{ E} =
\]

\[
A |0>|0> + B |1>|0> + C |0>|1> \quad \text{(16)}
\]

where \( A = x \quad B = -yr \quad C = yt \)

An Empty B_{\text{IN}} mode produces 3 possible B_{\text{OUT}} modes with amplitudes A, B and C.

\[
\text{Note that } A^2 + B^2 + C^2 = 1
\]
A Full input mode $B_F$ is transformed by the beam splitter into five "full output modes":

$$|B3\rangle|B4\rangle_F = |B5\rangle|B6\rangle_F =$$

$$D|2\rangle|0\rangle + E|1\rangle|1\rangle + F|0\rangle|2\rangle + G|1\rangle|0\rangle + H|0\rangle|1\rangle \quad (17)$$

where $D = -stry \quad E = (t^2 - r^2)y \quad F = stry \quad G = tx \quad H = rx$

A Full $B_{IN}$ Mode produces 5 possible $B_{OUT}$ modes with amplitudes $D$, $E$, $F$, $G$, and $H$

Note that $D^2 + E^2 + F^2 + G^2 + H^2 = 1$

The product of a Full $B_{OUT}$ mode and an Empty $B_{OUT}$ mode produces 15 possible outcomes.

Equations (16) and (17) are the keys to calculating the Kalamidas Effect. With these two equations we can calculate the probability amplitudes for all fifteen (15) possible responses of the Kalamidas Device.
Return to EQ (1). We replace the single \(|B>|s with the double \(|B>||B>|s we have calculated. For each of the two terms in EQ (1) we will obtain a similar expression:

\[ |A1>|A2> [|B3>|B4>] [|B5>|B6>] = 12[34][56] \]

the two terms differ only in which of the single \(|A>|s and which of the double \([|B>||B>|] brackets are empty or full.

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FIRST TERM IN THE ENTANGLEMENT
[full][empty][FULL][EMPTY]

SECOND TERM IN THE ENTANGLEMENT
[empty][full][EMPTY][FULL]

where small letters indicate the single-modes A1 and A2 and upper case letters indicate the double modes B3B4 and B5B6

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FIRST TERM IN THE ENTANGLEMENT

[FULL] x [EMPTY] = 5x3

E[11]
F[02]
G[10]
H[01]
From which we calculate all 15 possible B detector outcomes and their associated probability amplitudes.

\[
\begin{align*}
EA[1100] & \quad EB[1110] & \quad EC[1101] \\
FA[0200] & \quad FB[0210] & \quad FC[0201] \\
GA[1000] & \quad GB[1010] & \quad GC[1001] \\
HA[0100] & \quad HB[0110] & \quad HC[0101]
\end{align*}
\]

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SECOND TERM IN THE ENTANGLEMENT

[EMPTY] \times [FULL] = 3x5

B[10]
C[01]

From which we calculate all 15 possible B detector outcomes and their associated probability amplitudes.

\[
\begin{align*}
BD[1020] & \quad BE[1011] & \quad BF[1002] & \quad BG[1010] & \quad BH[1001] \\
CD[0120] & \quad CE[0111] & \quad CF[0102] & \quad CG[0110] & \quad CH[0101]
\end{align*}
\]

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Each of the square brackets represents one possible output of the Kalamidas Device. For each of these outcomes we can ask the following important question:
Question #1: From the form of this output, can we infer the nature of the input Fock state that produced it?

If the answer is **YES**, then that output preserves which-path information.

If the answer is **NO**, then that output has quantum-erased which-path information.

For eleven of these outputs, the answer to question #1 is **YES**: the nature of the input state can be inferred from the output state.

The fact that for four (4) of these outputs, the answer to question #1 is **NO** is the essence of the Kalamidas Effect. For these four outputs the nature of the input state **cannot** be inferred from the output. For these four outputs, the which-path information has been quantum-erased. The B photons responsible for these four outcomes have been successfully ambiguified.

Let's consider a few examples -- for instance the output state \([2000]\). Because a 2-photon output can only arise from a FULL input B state and a 0-photon output can only arise from an EMPTY input B state, we can infer that the output \([2000]\) arose from the input state \(|B1> |B2> = [10]|\).

Kalamidas points out that any \([xx]_B\) detector with a total of 2 counts could only have been produced by a FULL input state while any \([xx]_B\) with a total of zero counts could only be produced by an EMPTY input. However \([xx]_B\) outputs
with only one count could be produced by either a FULL or EMPTY state mixed with an appropriate gray-state input.

Therefore any total response which is constructed of two one-photon outputs is necessarily ambiguous. There are exactly four of these kinds of outputs.

These 4 output states (Kalamidas outputs) are:

\[
\begin{align*}
[1010] \\
[1001] \\
[0110] \\
[0101]
\end{align*}
\]

These four Kalamidas outputs possess ambiguous inputs because we cannot tell whether their single-photon output states arose from 1. a FULL input B state \(|1\rangle\) combined with an EMPTY gray state \(|0\rangle\); or from 2. an EMPTY input B state \(|0\rangle\) combined with a FULL gray state \(|1\rangle\).

Now comes the good part.

The discerning reader may have noticed that the fifteen output terms calculated for the \textbf{FIRST TERM OF THE ENTANGLEMENT} have very few elements in common with the fifteen output terms calculated for the \textbf{SECOND TERM OF THE ENTANGLEMENT}.

In fact, there are only four such elements common to both terms. And these four elements happen to be the four Kalamidas outputs.
If we call the 4 Kalamidas outputs TKOs and the remaining 11 outputs NKO (for non-Kalamidas outputs) we can rewrite the original two-photon entanglement EQ (1) as:

\[
|AB>_{KA} = s^{-1}[|A1> \exp(iQ \sum (11 \text{ NKO terms}) \\
+ |A2> \sum (11 \text{ different NKO terms}) \quad (18) \\
+ \{|A1>\exp(iQ + |A2>\} \{xyrt([K1] - [K2])\} \\
+|A1>\exp(iQ \{xytt[K3] - xyrr[K4]\} \\
+|A2> \{-xyrr[K3] + xytt[K4]\} ]
\]

Where the [K]s are the four "Kalamidas outputs":

\[
[K1] = [0101] \quad [K2] = [1010] \\
[K3] = [1001] \quad [K4] = [0110]
\]

EQ (18) is the main result of this paper. It represents the \textbf{probability amplitude} of the correlations between Alice's A photon taking paths |A1> and/or |A2> and the 26 distinguishable B outcomes of Bob's Kalamidas device—before Alice makes her choice as to how she wants to measure her A photons.

What EQ (18) shows is that 11 of the outcomes of Bob's detectors are correlated only with \textbf{one path} |A1> of Alice's photon, 11 other outcomes of Bob's detectors are correlated only with \textbf{one path} |A2> of Alice's photon while 4 outcomes of Bob's detectors (what I have called the
"TKO terms") are correlated in a complicated way with both paths (|A1> and |A2>) that Alice's photon can take from Source S.

Next we are going to calculate what happens when Alice makes a choices to measure her A photons using a simple interferometer in where her two original |A1> and |A2> paths are combined in a 50/50 beam splitter to produce two new output paths |A3> and |A4>.

From the form of EQ (18), we can anticipate what will happen at this beam splitter. Each NKO term will be added incoherently at Alice's splitter with a probability proportional to the sum of the squares of the amplitudes of the NKO terms. The expiQ factor will be unity and no interference will be observed.

On the other hand when the TKO part of this equation encounters Alice's splitter the two paths will be added coherently with a probability proportional to the sum of the squares of the amplitudes of the TKO terms. The expiQ factor will turn into sine and cosine factors which will modulate the probability of detecting photons at A3 and A4 as Alice changes her phase Q.

When Bob makes the Kalamidas choice, Alice will observe interference phenomena. When Bob makes the Fock choice, no interference will occur. Hence Bob's choice (K or F) can result in a signal transmitted instantly to Alice in the form of Interference or No Interference.

The action of a 50/50 beam splitter on Alice's A photons
looks like this (see Eq (7)):

\[
|A1> = s^{-1}[|A3> + |A4>]
\]
\[
|A2> = s^{-1}[-|A3> + |A4>]
\]

We apply the 50/50 beam splitter transformation to EQ (18), transforming Alice's states \(|A1>\) and \(|A2>\) into Alice's states \(|A3>\) and \(|A4>\).

\[
|AB>_{KA} =
\]

\[
\frac{1}{2} \left[ \text{expiQ} \sum (11 \text{ NKO terms}) \right.
\]
\[
- \sum (11 \text{ different NKO terms}) |A3> \\
\]
\[
+ \frac{1}{2} \left[ \text{expiQ} \sum (11 \text{ NKO terms}) \right.
\]
\[
+ \sum (11 \text{ different NKO terms}) |A4> \\
\]
\[
+ \frac{1}{2}xy \left[ \{Ert([K1] - [K2]) - rt([K1] - [K2]) \right. \\
+ E (tt[K3] - rr[K4]) \\
- (- rr[K3] + tt[K4]) \} |A3> \\
\]
\[
\left. + \{Ert([K1] - [K2]) + rt([K1] - [K2]) \right. \\
+ E (tt[K3] - rr[K4]) \\
- (rr[K3] - tt[K4]) \} |A4> \]
\]

where \(E = \text{expiQ}\)

EQ (19) represents \textbf{probability amplitudes} connecting Bob's outcomes with Alice's two outcomes \(|A3>\) and \(|A4>\).
To calculate the \textbf{probabilities} of these connections we multiple the bra $\langle AB \rangle_{KA}$ into ket $|AB\rangle_{KA}$ and apply Feynman's rule: add squares of amplitudes for distinguishable processes; add amplitudes and then square for distinguishable processes. Fortunately all of the terms in the above sum represent distinguishable processes -- each term in the sum represents a different pattern of photon responses in Bob's Kalamidas detector array $[B3B4B5B6]$. 

Multiplying bra $\langle AB \rangle_{KA}$ into ket $|AB\rangle_{KA}$ to calculate probabilities, we obtain (after much tedious algebra):

\begin{align*}
\langle ABIAB\rangle_{KA} &= 1/2 (x^4 + y^2)(\langle A3|A3\rangle + \langle A4|A4\rangle) \\
&+ 1/4 (xy)^2 \{2 (1 - \cos Q) r^2 t^2 ([K1]^2 + [K2]^2) \\
&+ (t^4 + r^4 + 2 r^2 t^2 \cos Q)([K3]^2 + [K4]^2)) \langle A3|A3\rangle \quad (20) \\
&+ 1/4 (xy)^2 \{2 (1 + \cos Q) r^2 t^2 ([K1]^2 + [K2]^2) \\
&+ (t^4 + r^4 - 2 r^2 t^2 \cos Q)([K3]^2 + [K4]^2)) \langle A4|A4\rangle\}
\end{align*}

Where $[K1]^2 = \langle 0101|0101\rangle$

for a typical Kalamidas output.
EQ (20) is the probability for Alice to observe photons at detectors A3 and A4 coincident with the 4 Kalamidas outcomes. All non-Kalamidas outcomes (NKOs) have been summed over. It is remarkable that this NKO sum which contained many complex functions of x, y, r and t (twenty-two terms squared and summed) reduces to such a simple factor: (x^4 + y^2). Also we note that this sum depends only on gray-light parameters (x,y) and is independent of the beam-splitter parameters (r, t).

The second and third terms of EQ (20) describe interference effects. We note that interference vanishes when the Kalamidas effect is "turned off", either by making the gray light "black" (x=1) or "white" (y = 1), or by removing the beam splitters (r = 0).

Gray-light parameters (x,y): The interference terms are maximized when x = y, that is, when the gray light consists of half real photon |1> and half vacuum |0>.

Beam-splitter parameters (r,t): The interference terms are maximized when r = t, that is, when Bob's beam spitters are 50/50.

One might suppose, from this gray-light dependence, that the Kalamidas Experiment could be used by Bob to send superluminal signals to Alice by toggling the gray light between its two extremes of effecting maximum interference at Alice's detectors: (x = y) to effecting none: (x = 1). This could be accomplished using a simple electro-optical shutter (Kerr cell).
Although EQ (20) describes interference terms at Alice's detectors, some of these terms (perhaps all) are still correlated with Bob's Kalamidas outputs.

For the Kalamidas Experiment to operate as a viable FTL signaling device, Alice's interference terms must not depend on coincidence counts at any of Bob's detectors. So, when we sum over all of Bob's Kalamidas outcomes, any interference terms that remain will represent a Bob-to-Alice superluminal signal.

Summing over all four Kalamidas outcomes we obtain:

\[
<AB|AB>_{KA} = \frac{1}{2} (x^4 + y^2 + (xy)^2) (<A3|A3> + <A4|A4>)
\]

\[
= \frac{1}{2} (<A3|A3> + <A4|A4>)
\]

(21)

No superluminal signal survives this summation. The Kalamidas choice, like the Frost choice, produces interference in Alice's detectors only when coincidence-triggered by results at Bob's Kalamidas outputs. Thus the Kalamidas experiment (at least in Nick Herbert's realization) fails as an FTL signaling scheme.

CONCLUSIONS

1. Recently Demetrios Kalamidas proposed an original scheme to send signals faster-than-light from Bob to Alice
using quantum entanglement. His proposal depends on a new way to erase Bob's which-path info by "ambiguifying" Bob's number-certain Fock states by mixing these states in a beam splitter with number-uncertain light.

2. In Kalamidas's original proposal (http://lanl.arxiv.org/abs/1110.4629) Bob quantum-erases Alice's distant "which-path" information by mixing his B photons with a state of uncertain photon number. For this number-uncertain state, Kalamidas chose a weak coherent state. In my version of the Kalamidas effect, I employ a "gray state", the most general 1-photon uncertain-number state, for the same purpose.

3. Nick Herbert's version of the Kalamidas experiment conspicuously fails to operate as an FTL signaling device. The Kalamidas choice is seen to be a variant of what I have called the Frost choice, that is, a choice by Bob that induces interference in Alice's detectors, but interference of a sort that is invisible until revealed by coincidence signals Bob sends to Alice by conventional channels.

4. This refutation of Nick's version casts some doubt on Kalamidas's original claim to have discovered a way for Bob to cause interference at Alice's detectors without coincidence triggering. Since the present scheme seems more general than Kalamidas's original proposal, his original proposal must contain some flaw.

5. I suspect that this flaw is located in Kalamidas's description of his number-uncertain state. Kalamidas uses
a weak coherent state mixed with B's photons in "barely reflecting \( r \ll 1 \) beam splitters. His weak coherent state has the form, after the beam splitter, of \( (1 + r \alpha a^\dagger) |0\rangle \). He claims that "terms proportional to \( r\alpha \) are significant because \( \alpha \) can be arbitrarily large in magnitude". Now it is certainly true that the magnitude \( \alpha \) of an ordinary coherent state can be made arbitrarily large, but a "truncated coherent state" consisting only of \(|0\rangle\) and \(|1\rangle\) kets is severely limited by normalization requirements—which I express in my version of this scheme as \( x^2 + y^2 = 1 \). I suspect that, when Kalamidas clarifies his scheme by inserting a realistic two-mode state as input to his beam splitters, then his FTL proposal will fail as completely as my own.

6. I wish to congratulate Demetrios Kalamidas for coming up with his imaginative new FTL scheme which gave me much pleasure and excitement to analyze. I would also like to thank him for correcting an error in my work which, up until his intervention, seemed to show confirmation of his FTL signaling claim by producing a large interference effect at Alice's detectors with a magnitude of 25%. After his timely input, the present (presumably correct) calculation demonstrates a complete refutation of any FTL claim. However, the Kalamidas scheme of erasing which-path info by mixing Fock light with gray light is clever and may yet find new technical applications in areas other than superluminal communication technology.
REFERENCES


**NICK HERBERT** (quanta@cruzio.com)
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