FTL SIGNALING MADE EASY: MAXIMIZING THE KALAMIDAS EFFECT

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Abstract: I rederive the Kalamidas effect (a purported FTL signaling scheme) using familiar conventions and generalize his ingenious proposal to achieve fringe visibility of 22%.

Recently Demetrios Kalamidas published a purported FTL signaling scheme (Kalamidas 2013) which is clever but hard to understand due to a difficult-to-read choice of naming conventions. So that his ingenious experiment may be more widely appreciated, I reproduce his proof, using more obvious (to me) notation.

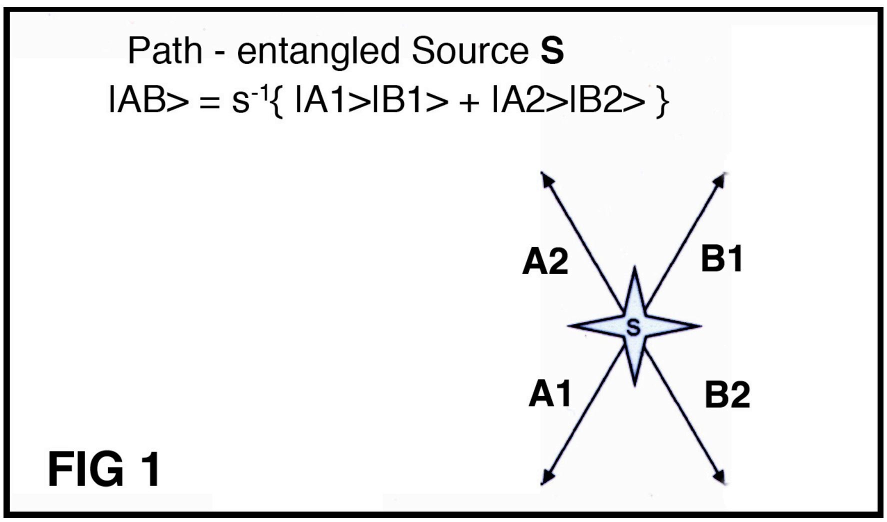


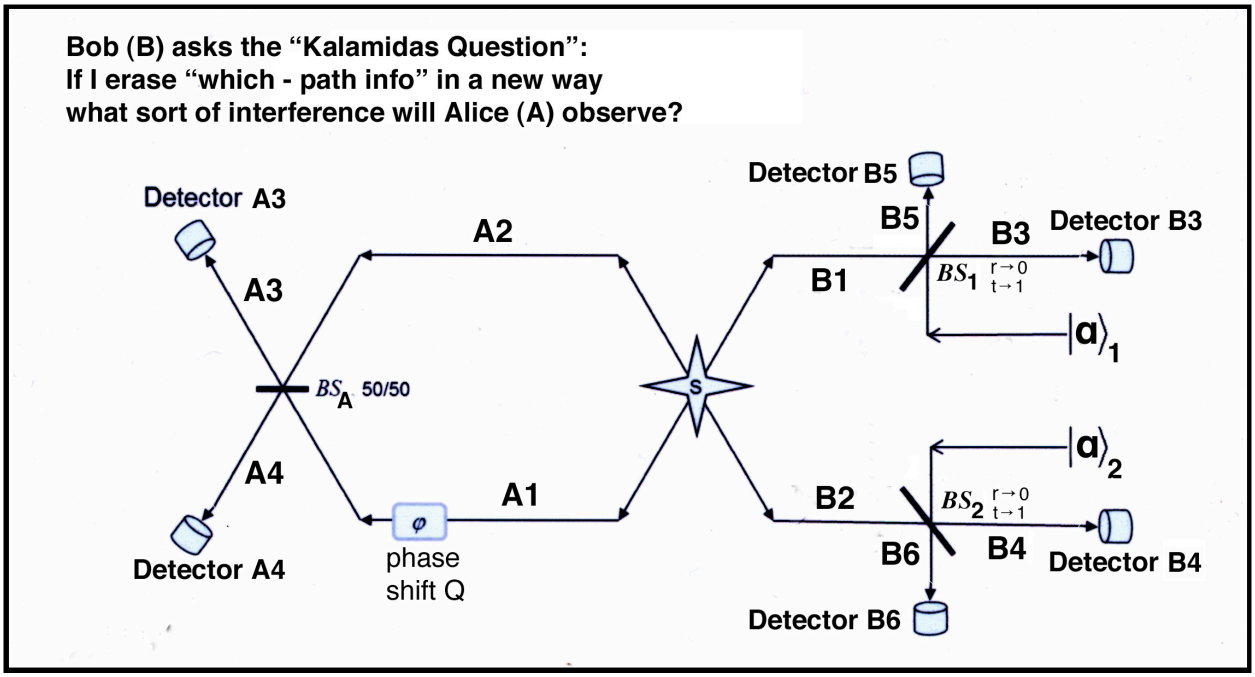
FIG 1

The proof begins by considering a photon source S that emits a pair of photons A (to Alice) and B (to Bob) in opposite directions. Each photon pair is prepared in a superposition of two possible paths which are labeled 1 and 2. The wave function **Ψ** (A, B) for this path-entangled-source is given by:

$$\Psi (A, B) = s^{-1}\{ |A_1\rangle|B_1\rangle + |A_2\rangle|B_2\rangle\} \qquad (1)$$

where $s = \sqrt{2}$.

Although both photons are path-superposed, Bob can break that superposition by measuring either B_1 or B_2 thus collapsing his B photon to a single path. Because of their mutual entanglement, Alice's photon also (instantly?) collapses to a single path. When Alice's photon travels both paths (1 and 2), there is the possibility of her detecting interference; when Alice's photon travels only one path, interference is impossible. The Kalamidas Effect works by using a novel way of erasing Bob's "which-path info" and hence distantly producing or suppressing interference at Alice's detectors.



In this illustration of the Kalamidas Experiment, Alice has deployed a simple interferometer consisting of a variable phase-shifter Q plus a 50/50 beam splitter. If all or some of Alice's A photons "take both paths", then Alice will observe a sinusoidal variation of counting intensities at her photon detectors A_3 and A_4.

The job of Bob is to create interference at Alice's detectors by erasing "which-path info" of his B photons. He does this by mixing each B photon path with a **weak coherent state α** in such a manner as to partially ambiguate which path his photon took.

A coherent state α is mixed into each of Bob's two possible photon paths via a **highly transparent beam splitter** ($|t|$ ---> 1, $|r|$ ---> 0).

Consider Bob's upper path B_1, Since beam splitter BS_1 is highly transparent, most of Bob's B_1 photons end up in detector B_3. Only a few will be reflected into detector B_5. Note that I have chosen the numbering convention so that anything having to do with Bob's path B_1 is labeled with an **odd number**. Consequently the coherent state that Bob mixes with B_1 at this beam splitter is labeled α_1.

Following this convention, Bob's photon path B_2 enters beam splitter BS_2 where it splits mostly into photon detector B_4 and rarely into B_6. Likewise the coherent state that mixes with photon B_2 is labeled α_2.

My beam splitters operate according to the following convention:

$$a(B_1)^\dagger \text{ ---> } t a_3^\dagger + r a_5^\dagger \quad (2)$$

$$(\alpha)_1^\dagger \text{ ---> } (-r\alpha)_3^\dagger (t\alpha)_5^\dagger \quad (3)$$

where $a(B_1)^\dagger$ and $(\alpha_1)^\dagger$ are the photon creation operators for Bob's input photon B_1 and for the input coherent state α_1. And $a_3^\dagger$ and $a_5^\dagger$ are the photon creation operators for output modes 3 and 5.

$$a_3^\dagger|0\rangle = |1\rangle_3 \text{ and } (a_3^\dagger|n\rangle_3 = \sqrt{(n+1)}|(n+1)\rangle_3 \quad (4)$$

$$(\alpha_1)^\dagger|0\rangle = |(\alpha_1)\rangle \text{ ---> } (-r\alpha)_3^\dagger (t\alpha)_5^\dagger|0\rangle = |(-r\alpha)\rangle_3 \; |(t\alpha)\rangle_5 \quad (5)$$

Equations (3) and (5) express the unusual fact that a coherent state splits at a beam splitter into two output coherent states. No other form of light splits so cleanly but inevitably ends up entangled. Proof of this unique property of coherent states is due to Aharonov and is routinely assigned as a problem in modern optics texts. The transmission and reflection amplitudes are real and normalized -- $t^2 + r^2 = 1$. A similar set of beam splitter equations exists for Bob's "even beam" B_2.

Bob intends to erase which-path info by mixing a significant number of photons from the coherent state α into paths B_3 and B_4. Since the reflection coefficient r is small, the amplitude α of the coherent state must be correspondingly large so that the product $r\alpha$ is close to unity. In his original paper, Kalamidas chose α so that the reflected coherent state becomes a **weak coherent state** containing only Fock states |0> and |1>.

$$|(-r\alpha)>_3 \dashrightarrow |0>_3 - r\alpha|1>_3 \qquad (6)$$

In practice this means that $r\alpha$ is between 0.1 and 0.3. When mode B_1 is empty (= no input photon), the amplitude at output modes 3 and 5 is due solely to the coherent state input and has the form:

$$B_1(\text{empty}) \equiv |0>_1 \dashrightarrow (1 - r\alpha_1 a^\dagger)_3|0>_3\ |(t\alpha_1)>_5 \qquad (7)$$

$$|0>_1 \dashrightarrow (|0>_3 - r\alpha_1|1>_3)\ |(t\alpha_1)>_5 \qquad (8)$$

Repeating this calculation for the "even mode" B_2 we obtain the following expression for the two empty modes:

$$|0>_1 \dashrightarrow (|0>_3 - r\alpha_1|1>_3)\ |(t\alpha_1)>_5 \qquad (9)$$

$$|0>_2 \dashrightarrow (|0>_4 - r\alpha_2|1>_4)\ |(t\alpha_2)>_6 \qquad (10)$$

A slightly more complicated calculation yields for B_1(full) the expression:

$$B_1(\text{full}) \equiv |1>_1 \dashrightarrow \{ta^\dagger - tr\alpha_1 a^\dagger a^\dagger\}_3\, |0>_3\, |(t\alpha_1)>_5$$
$$+\ \underline{r\,(1 - r(1 - r\alpha_1 a_3{}^\dagger)|0>_3 a_5{}^\dagger |(t\alpha_1)>_5} \qquad (11)$$

The second (underlined) term is linear in r. Since r ---> 0 while $r\alpha$ is finite, this term is small compared to terms prefaced by factors of $r\alpha$.In all further calculation we omit this underlined term.

Repeating this calculation for the even mode B_2 we obtain:

$$|1>_1 \dashrightarrow \{t|1>_3 - str\alpha_1|2>_3\}\, |(t\alpha_1)>_5 \qquad (12)$$
$$|1>_2 \dashrightarrow \{t|1>_4 - str\alpha_2|2>_4\}\, |(t\alpha_2)>_6 \qquad (13)$$

Now we put both these calculations together and compute the photon amplitude of Bob's four detectors $B_3B_4B_5B_6$ when there is one photon in input path B_1 and zero photons in input path B_2. We want to calculate $B_1(\text{full})B_2(\text{empty}) = |1>_1|0>_2$. After a bit of algebra this becomes:

$$|1>_1|0>_2 = \{t|1>_3|0>_4 - \mathbf{rt\alpha_2\, |1>_3|1>_4}$$
$$- str\alpha_1|2>_3\, |0>_4 + str^2\alpha_1\alpha_2|2>_3|1>_4\}\,\{\,\underline{|}(t\alpha_1)>_5|(t\alpha_2)>_6\,\} \qquad (14)$$

$$|0>_1|1>_2 = \{t|0>_3|1>_4 - \mathbf{rt\alpha_1\, |1>_3|1>_4}$$
$$- str\alpha_2|0>_3\, |2>_4 + str^2\alpha_1\alpha_2|1>_3|2>_4\}\,\{\,\underline{|}(t\alpha_1)>_5|(t\alpha_2)>_6\,\} \qquad (15)$$

Equation (14) represents the amplitudes of the 4 different possibilities of path B_1 and Equation (15) represents the amplitudes of the 4 different possibilities of path B_2. Three of these possibilities represent distinct paths but the term indicated in bold type has the same outcome for both paths -- hence represents a superposition of both paths. The presence of this term $\mathbf{rt\alpha\, |1>_3|1>_4\, \underline{|}(t\alpha_1)>_5|(t\alpha_2)>_6}$ will lead to interference at Alice's detectors.

As pointed out by Kalamidas, three of these outcomes uniquely specify which path the input photon took, but the $|1>_3|1>_4$ outcome bears no trace of its origin. Either the input photon took B_1 path and triggered B_3 while the α_2 photon triggered B_4. Or the input photon took B_2 path and triggered B_4 while the α_1 photon triggered B_1. Path information has been erased and the $|1>_3|1>_4$ outcome represents **a coherent both-path event**.

Due to the appearance of the path-ambiguous $|1>_3|1>_4$ outcome it surely appears as if the Kalamidas scheme succeeds in erasing (at least partially) which-path information about Bob's photon **without using coincidence counting**. And via quantum entanglement the corresponding which-path info about Alice's photon must also be erased.

This distant Alice erasure is under Bob's control because he can decide whether or not to mix his coherent states with the B inputs. Consequently the Kalamidas Effect will operate as a **de facto** FTL signaling device.

From Eqs (14) and (15) we can calculate the intensity I(Q) by employing the familiar Feynman rule: one-path amplitudes: square before adding; both-path amplitudes: add before squaring.

There are 3 one-path amplitudes $\{ 1, sr\alpha, s(r\alpha)^2 \}$ and one both-path amplitude $\{ r\alpha \}$, where we have set $t = 1$ and $\alpha_1 = \alpha_2 = \alpha$. In one of the paths we place a phase factor 1; in the other a phase factor exp iQ.

$$I(Q) = 2 + 4K + 4K^2 + 4K\cos^2(Q/2) \qquad (16)$$

where K is the "Kalamidas factor" $(r\alpha)^2$.

Fringe visibility V is defined as:

$$V = (I_{max} - I_{min})/(I_{max} + I_{min}) \qquad (17)$$

$$V_K = K/(1 + 2K + 2K^2) \qquad (18)$$

In the weak-coherent state approximation rα lies somewhere between 0.1 and 0.3. From EQ (18) this gives a fringe visibility somewhere between 1% and 9%. Thus the FTL signal in this approximation seems quite substantial.

But suppose we increase rα to include a few more Fock states in the coherent state truncation. We will call this a "four-Fock coherent state". When rα = 0.6, a four-Fock state is achieved in which the fifth Fock state has a magnitude of only 2%. Will this larger state increase or decrease the fringe visibility?

$$|(-r\alpha)\rangle_3 \dashrightarrow W \{ |0\rangle_3 - r\alpha|1\rangle_3 + s^{-1}(r\alpha)^2|2\rangle_3 - z^{-1}(r\alpha)^3 |3\rangle_3\} \qquad (19)$$

where $s = \sqrt{2}$, $z = \sqrt{6}$ and $W = \exp(-|r\alpha|^2/2)$.

The empty state |0> expands thus:

$$|0\rangle_1 \dashrightarrow W \{ |0\rangle_3 - r\alpha|1\rangle_3 + s^{-1}(r\alpha)^2|2\rangle_3 - z^{-1}(r\alpha)^3 |3\rangle_3\} |(t\alpha_1)\rangle_5 \quad (20)$$

and the full state |1> now has four terms instead of two:

$$|1\rangle_1 \dashrightarrow W \{ |1\rangle_3 - sr\alpha|2\rangle_3 + 1/2\, z\, (r\alpha)^2|3\rangle_3 - 2\, z^{-1}(r\alpha)^3 |4\rangle_3\} |(t\alpha_1)\rangle_5 \qquad (21)$$

There are now sixteen path amplitudes, 13 of which are one-path and three of which are both-path, namely |1>|1>, |2>|2> and |3>|3>.These three amplitudes contribute to which-path erasure. Each of these three both-path amplitudes has the same sign so they will all add constructively to create interference fringes. The 13 one-path amplitudes will add incoherently and act to diminish fringe visibility. For the four-Fock coherent state, who will win?-- the 13 incoherent states (noise) or the 3 coherent states (signal)?

Keeping only terms in K and K^2 we calculate the intensity $I(Q)_{K4}$ for the four-Fock Kalamidas effect:

$$I(Q)_{K4} = W^2 \{2 + 4K + 8K^2 + 4(K + 2K^2)\cos^2(Q/2)\} \quad (22)$$

And the fringe visibility for the four-Fock Kalamidas effect is:

$$V_{K4} = (K+ 2K^2)/(1 + 3K + 6K^2) \quad (23)$$

For $r\alpha = 0.6$ this visibility becomes $V_{K4} = 22\%$, a quite substantial improvement over the two-Fock Kalamidas effect ($V_{K2} = 9\%$). One might reasonably guess, that by including even more Fock states in the path-erasing coherent state, fringe visibilities as high as 30% might be achieved. In his JOSA paper, Kalamidas claims that his quantum erasure scheme "will obtain **low visibility** single-photon interference" without coincidence detection. My calculations show that in this respect Kalamidas was unduly pessimistic--the fringe visibilities one might reasonably hope to achieve in the Kalamidas experiment are quite substantial.

Conclusion: Despite much effort, I have been unable to refute Demetrios Kalamidas's ingenious scheme for Bob to transmit signals to Alice at superluminal speed. However, I have reformulated the Kalamidas proposal in (perhaps) more congenial notation for others to attempt to refute. Also I have calculated the effect size of Kalamidas's original proposal (9%) and devised an extension of the Kalamidas experiment which achieves an effect size of 22%.

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Reference: Kalamidas, D. **Journal of the Optical Society of America B, Volume 30 May 2013, page 1290.** Original ArXiv preprint at: http://lanl.arxiv.org/abs/1110.4629